Exercise 21

Find the intersection of the planes $x + (y - 1) + z = 0$ and $-x + (y + 1) - z = 0$.

Solution

The intersection for two planes is a straight line, which can be parameterized as

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where $\mathbf{m}$ is the direction vector and $\mathbf{b}$ is the position vector for any point on the line. The normal vectors to the given planes are obtained from the coefficients of x, y, and z: $(1, 1, 1)$ and $(-1, 1, -1)$. Take the cross product of these two to find the direction vector of the line.

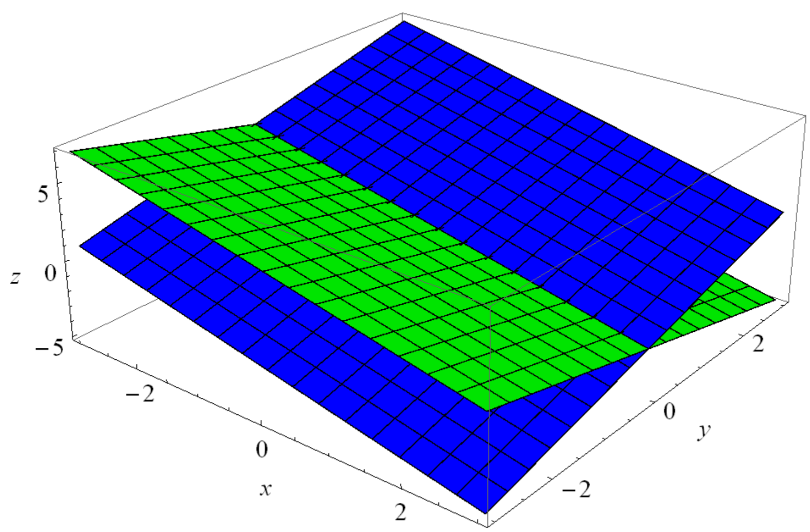
$$\mathbf{m} = (1,1,1)\times(-1,1,-1) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = (-1-1)\hat{\mathbf{x}} - (-1+1)\hat{\mathbf{y}} + (1+1)\hat{\mathbf{z}} = -2\hat{\mathbf{x}} + 2\hat{\mathbf{z}} = (-2, 0, 2)$$

All that's left is to find a point common to both planes.

$$\left.\begin{aligned} x + (y-1) + z = 0 \\ -x + (y+1) - z = 0 \end{aligned}\right\} \quad \rightarrow \quad 2y = 0 \quad \rightarrow \quad y = 0$$

Choose $x = 1$, $y = 0$, and $z = 0$, for example. Then $\mathbf{b} = (1, 0, 0)$, and the line is

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{m}t + \mathbf{b} \\ &= (-2, 0, 2)t + (1, 0, 0) \\ &= (-2t + 1, 0, 2t). \end{aligned}$$



In green is $x + (y - 1) + z = 0$, and in blue is $-x + (y + 1) - z = 0$.

This answer is in disagreement with the one at the back of the book. $x = t$, $y = 2t$, and $z = -5t$ satisfies neither $x + (y - 1) + z = 0$ nor $-x + (y + 1) - z = 0$, but it does satisfy the equations in Exercise 20.